Quantum noise, entanglement and chaos in the quantum field theory of mind/brain states

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Abstract We review the dissipative quantum model of brain and present recent developments related with the rôle of entanglement, quantum noise and chaos in the model.

1 Introduction

In this paper we present a short review of the dissipative quantum model of brain (Vitiello, 1995; 2001) and discuss some of its recent developments (Pessa and Vitiello, 2003) related with the rôle of entanglement, quantum noise and chaos in the model.

The quantum model of brain was originally formulated by Umezawa and Ricciardi (Ricciardi and Umezawa, 1967) and subsequently developed by Stuart, Takahashi and Umezawa (Stuart, Takahashi and Umezawa, 1978; 1979) by Jibu and Yasue (Jibu and Yasue, 1995) and by Jibu, Pribram and Yasue (Jibu, Pribram and Yasue, 1996). The formalism is the one of the quantum field theory (QFT). The extension of the model to the dissipative dynamics has been worked out more recently (Vitiello, 1995) (see also (Alfinito and Vitiello, 2000; Pessa and Vitiello, 1999)) and a general account in given in the book by G.V. My Double unveiled (Vitiello, 2001).

The motivations at the basis of the formulation of the quantum model of brain by Umezawa and Ricciardi trace back to the laboratory observations leading, since the 1940's, Lashley to remark that "masses of excitations... within general fields of activity, without regard to particular nerve cells" (Lashley, 1942; Pribram, 1991) are involved in the determination of behavior. In the middle of the 1960's Karl Pribram, also motivated by experimental observations, started to formulate his holographic hypothesis. Information appears indeed in such observations to be spatially uniform "in much the way that the information density is uniform in a hologram" (Freeman, 1990; 2000). While the activity of the single neuron is experimentally observed in form of discrete and stochastic pulse trains and point processes, the "macroscopic" activity of large assembly of neurons appears to be spatially coherent and highly structured in phase and amplitude (Freeman, 1996; 2000).

Umezawa and Ricciardi, motivated thus by such an experimental situation, formulated in 1967 (Ricciardi and Umezawa, 1967) the quantum model of brain as a many-body physics problem, namely by using the formalism successfully tested in condensed matter experiments, the QFT with spontaneous breakdown of symmetry. Such a formalism provides indeed the only available theoretical tool capable to describe long range correlations such the ones observed in the brain presenting almost simultaneous responses in several regions to some external stimuli. As a matter of fact, the understanding of such correlations in terms of modern biochemical and electrochemical processes is still lacking, which suggests that these responses could not be explained in terms of single neuron activity (Pribram 1971; 1991).

In QFT the dynamics (i.e. the Lagrangian) is in general invariant under some group, say G, of continuous transformations. Spontaneous breakdown of symmetry occurs when the

minimum energy state (the ground state or vacuum) of the system is not invariant under the full group G, but under one of its subgroups. Then it can be shown (Itzykson and Zuber, 1980; Umezawa, 1993) that collective modes, the so-called Nambu-Goldstone (NG) boson modes, are dynamically generated. Propagating over the whole system, these modes are the carriers of the ordering information ($long\ range\ correlation$): order manifests itself as a global property dynamically generated. The long range correlation modes are responsible for keeping the ordered pattern: they are coherently condensed in the ground state. In the crystal case, for example, they keep the atoms trapped in their lattice sites. The long range correlation thus forms a sort of net, extending over all the system volume, which traps the system components in the ordered pattern. This explain the macroscopic collective behavior of the components as a "whole".

The observable specifying the ordered state is called the order parameter. It is a measure of the condensation of the Nambu–Goldstone modes in the ground state and acts as a macroscopic variable: it may be considered to be the *code* specifying that specific ordered state.

It is to be remarked that the spontaneous breakdown of symmetry is possible since in QFT there exist infinitely many ground states or vacua which are physically distinct (technically speaking, they are "unitarily inequivalent"). In quantum mechanics (QM), on the contrary, all the vacua are physically equivalent and thus there cannot be symmetry breakdown.

In the following section we will see how these features of QFT apply to the brain model. The paper is organized as follows: in Section 2 we shortly summarize the main aspects of the quantum brain model. In Section 3 we consider its extension to the dissipative dynamics and comment on the brain–environment entanglement. In Section 4 and 5 we present the very recent developments concerning quantum noise and chaos, respectively. Section 6 is devoted to concluding remarks.

2 The quantum model of brain

In this Section we present a short summary of the Ricciardi and Umezawa model, closely following the ref: (Vitiello, 2001).

An essential, first requirement in the model is that stimuli coming to the brain from the external world should be coded and their effects on the brain should persist also after they have ceased; this means that stimuli should be able to change the state of the brain pre–existing the stimulation into another state where the information is "printed" in a stable fashion. This implies that the state where information is recorded under the action of the stimuli must be a ground state in order to realize the stability of the recorded information; and that symmetry is broken in order to allow the coding of the information. Recording of information is thus represented by coherent condensation of NG bosons implied by the symmetry breakdown. The NG collective modes are massless bosons, and thus their condensation in the vacuum does not add energy to it: the stability of the ordering, and therefore of the registered information, is thus insured.

The order parameter is specific to the kind of symmetry of the dynamics and its value is considered to be the *code* specifying the information printed in that ordered vacuum. Non–local properties, related to a code specifying the system state, are dynamical features of quantum origin: it is in this way that the stable and diffuse, non–local character of memory is represented in the quantum model; it is derived as a dynamical feature rather than as a property of specific neural nets (which would be critically damaged by local destructive actions).

The mechanism of recall of the stored information is related to the possibility of exciting collective modes out of the ground state. Suppose that an ordered pattern was printed on

the brain by condensation mechanism in the vacuum which was induced by certain external stimuli. Though an order is stored, brain is not conscious of this because it is in the ground state. However, when a similar external stimulation comes in, it easily excites the massless boson associated with the long range correlation. Since the boson is massless, any small amount of energy can cause its excitation. During the time of excitation, brain becomes conscious of the stored order (memory). This explains recollection mechanism (Ricciardi and Umezawa, 1967). The excited modes have finite life—time and thus the recall mechanism is a temporary activity of the brain, according indeed to our common experience. This also suggests that the capability to be "alert" or "aware" or to keep our "attention" focused on certain subjects (information) for a short or a long time may have to do with the short or long life—time of the excited modes out of the brain ground state.

It may also happen that under the action of external stimuli the brain may be put into an excited state, i.e. a quasi-stationary state of greater energy than the one of the ground state. Such an excited state also carries collective modes in their non-minimum energy state. Thus this state also can support recording some information. However, due to its higher energy such a state and the collective modes are not stable and will sooner or later decay: short-term memory is then modelled by the condensation of long range correlation modes in the excited states. Different types of short-term memory are represented by different excitation levels in the brain state.

For a further analysis of the short–term memory mechanism in terms of non–equilibrium phase transitions see also: (Sivakami and Srinivasan, 1983).

The brain model should explain how memory remains stable and well protected within a highly excited system, as indeed the brain is. Such a "stability" must be realized in spite of the permanent electrochemical activity and the continual response to external stimulation. The electrochemical activity must also, of course, be coupled to the correlation modes which are triggered by external stimuli. It is indeed the electrochemical activity observed by neurophysiology that provides (Stuart, Takahashi and Umezawa, 1978; 1979) a first response to external stimuli.

This has suggested to model the memory mechanism as a separate mechanism from the electrochemical processes of neuro-synapic dynamics: the brain is then a "mixed" system involving two separate but interacting levels. The memory level is a quantum dynamical level, the electrochemical activity is at a classical level. The interaction between the two dynamical levels is possible because of the specificity of the quantum dynamics: the memory state is a macroscopic quantum state due to the coherence of the correlation modes.

The problem of the coupling between the quantum dynamical level and the classical electrochemical level is then reduced to the problem of the coupling of two macroscopic entities. Such a coupling is analogous to the coupling between classical acoustic waves and phonons in crystals. Acoustic waves are classical waves; phonons are quantum NG long range modes. Nevertheless, their coupling is possible since the macroscopic behavior of the crystal "resides" in the phonon modes, so that the coupling acoustic waves-phonon is equivalently expressed as the coupling acoustic wave-crystal (which is a perfectly acceptable coupling from a classical point of view).

We remark that the quantum variables in the quantum model of brain are basic field variables (the electrical dipole field) and the brain is described as a macroscopic quantum system. Stuart, Takahashi and Umezawa (Stuart, Takahashi and Umezawa, 1978) have indeed remarked that "it is difficult to consider neurons as quantum objects". In other models of brain the relevant variables are binary variables describing the neuron's on/off activity. However, in the quantum model "we do not intend", Ricciardi and Umezawa say "to consider necessarily the neurons as the fundamental units of the brain".

The quantum model of brain fits the neurophysiological observations of memory nonlocality and stability. However, several problems are left open. One is that of memory capacity,

the overprinting problem: Suppose a specific code corresponding to a specific information has been printed in the vacuum. The brain then sets in that state and successive recording of a new, distinct (i.e. of different code) information, under the action of a subsequent external stimulus, is possible only through a new condensation process, corresponding to the new code. This last condensation will superimpose itself on the former one (overprinting), thus destroying the first registered information.

It has been shown (Vitiello, 1995) that by taking into account the dissipative character of brain dynamics may solve the problem of memory capacity and quantum dissipation also turns out to be crucial for the understanding of other functional features of the brain. In the next Section we present a short summary of the dissipative quantum model of brain.

3 The dissipative quantum model of brain

In the quantum model of brain the symmetry which undergoes spontaneous breakdown under the action of the external stimuli is the electrical dipole rotational symmetry. Water and other biochemical molecules entering the brain activity are, indeed, all characterized by a specific electrical dipole which strongly constrains their chemical and physical behavior. Once the dipole rotational symmetry has been broken (and information has thus been recorded), then, as a consequence, time-reversal symmetry is also broken: Before the information recording process, the brain can in principle be in anyone of the infinitely many (unitarily inequivalent) vacua. After information has been recorded, the brain state is completely determined and the brain cannot be brought to the state configuration in which it was before the information printing occurred. This is in fact the meaning of the well known warning ...NOW you know it!..., which tells you that since now you know, you are another person, not the same one as before...Once you have known, you cannot go back in time.

Thus, "getting information" introduces the arrow of time into brain dynamics; it introduces a partition in the time evolution, the distinction between the past and the future, a distinction which did not exist before the information recording. In other words, it introduce irreversibility, i.e. dissipation. The brain is thus, unavoidably, an open system.

When the system under study is not an isolated system it is customary in quantum theory to incorporate in the treatment also the other systems (which constitute the environment) to which the original system is coupled. The full set of systems then behaves as a single isolated (closed) one. At the end of the required computations, one extracts the information regarding the evolution of the original system by neglecting the changes in the remaining systems.

In many cases, the specific details of the coupling of our system with the environment may be very intricate and changeable so that they are difficult to be measured and known. One possible strategy is to average the effects of the coupling and represent them, at some degree of accuracy, by means of some "effective" interaction. Another possibility is to take into account the environmental influence on the system by a suitable *choice* of the vacuum state (the minimum energy state or ground state). The chosen vacuum thus carries the *signature* of the reciprocal system—environment influence at a given time under given boundary conditions. A change in the system—environment reciprocal influence then would correspond to a change in the choice of the system vacuum: the system ground state evolution or "story" is thus the story of the trade of the system with its environment. The theory should then provide the equations describing the system evolution "through the vacua", each vacuum corresponding to the system ground state at each time of its history.

In conclusion, in order to describe open quantum systems first of all one needs to use QFT (Quantum Mechanics does not have the many "inequivalent" vacua!). Then one also needs to use the time variable as a label for the set of ground states of the system (Celeghini, Rasetti and Vitiello, 1990): as the time (the label value) changes, the system moves to a

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"new" (physically inequivalent) ground state (assuming continuous changes in the boundary conditions determining the system–environment coupling). Here, "physically inequivalent" means that the system observables, such as the system energy, assume different values in different inequivalent vacua, as is expected to happen in the case of open systems.

One thus gets a description for the open systems which is similar to a collection of photograms: each photogram represents the "picture" of our system at a given instant of time (a specific time label value). Putting together these photograms in "temporal order" one gets a movie, i.e. the story (the evolution) of our open system, which includes the systemenvironment interaction effects.

The evolution of the \mathcal{N} -coded memory can be represented as a trajectory of given initial condition running over time-dependent states $|0(t)\rangle_{\mathcal{N}}$, each one minimizing the free energy functional. Recent results (Pessa and Vitiello, 2003; Vitiello, 2003) show that such trajectories may have chaotic character. We will discuss this in Section 5.

The mathematical representation of the environment must explicitly satisfy the requirement that the energy lost by the system must match the energy gained by the environment, and viceversa. All other details of the system–environment interaction may be taken into account by the vacuum structure of the system, in the sense above explained. Then the environment may be represented in the simplest way one likes, provided the energy flux balance is preserved. One possible choice is to represent the environment as the "time–reversed copy" of the system: time must be reversed since the energy "dissipated" by the system is "gained" by environment.

In conclusion, the environment may be mathematically represented as the *time-reversed image* of the system, i.e. as the system "double". What the system loses, the environment gains, and viceversa.

Let A_{κ} denotes the dipole wave quantum (dwq) mode, namely the Nambu–Goldstone mode associated to the spontaneous breakdown of rotational electrical dipole symmetry. \tilde{A}_{κ} will denote its "doubled mode". The \tilde{A} mode is the "time–reversed mirror image" of the A mode and represents the environment mode. Let $\mathcal{N}_{A_{\kappa}}$ and $\mathcal{N}_{\tilde{A}_{\kappa}}$ denote the number of A_{κ} modes and \tilde{A}_{κ} modes, respectively. The suffix κ here generically denotes kinematical variables (e.g. spatial momentum) or intrinsic variables of the fields fully specifying the field degrees of freedom.

Notice that the "tilde" or doubled mode is not just a mathematical fiction. It corresponds to a real excitation mode (quasiparticle) living in the system as an effect of its interaction with the environment: the couples $A_k\tilde{A}_k$ represent the correlation modes dynamically created in the system as a response to the system–environment reciprocal influence. It is the interaction between tilde and non–tilde modes that controls the time evolution of the system: the collective modes $A_k\tilde{A}_k$ are confined to live in the system. They vanish as soon as the links between the system and the environment are cut.

In the following Section we will see how these doubled modes may be understood in terms of Wigner functions and how they are related to quantum noise (Srivastava, Vitiello and Widom, 1995).

Taking into account dissipation requires (Vitiello, 1995) that the memory state, identified with the vacuum $|0>_{\mathcal{N}}$, is a condensate of equal number of A_{κ} and \tilde{A}_{κ} modes, for any κ : such a requirement ensures that the flow of the energy exchanged between the system and the environment is balanced. Thus, the difference between the number of tilde and non-tilde modes must be zero: $\mathcal{N}_{A_{\kappa}} - \mathcal{N}_{\tilde{A}_{\kappa}} = 0$, for any κ .

The label \mathcal{N} in the vacuum symbol $|0\rangle_{\mathcal{N}}$ specifies the set of integers $\{\mathcal{N}_{A_{\kappa}}, \text{ for any } \kappa\}$ which indeed defines the "initial value" of the condensate, namely the *code* associated to the information recorded at time $t_0 = 0$.

Note now that the requirement $\mathcal{N}_{A_{\kappa}} - \mathcal{N}_{\tilde{A}_{\kappa}} = 0$, for any κ , does not uniquely fix the set $\{\mathcal{N}_{A_{\kappa}}, \text{ for any } \kappa\}$. Also $|0>_{\mathcal{N}'} \text{ with } \mathcal{N}' \equiv \{\mathcal{N}'_{A_{\kappa}}; \mathcal{N}'_{A_{\kappa}} - \mathcal{N}'_{\tilde{A}_{\kappa}} = 0, \text{ for any } \kappa\}$ ensures the

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energy flow balance and therefore also $|0>_{\mathcal{N}'}$ is an available memory state: it will correspond, however, to a different code (i.e. \mathcal{N}') and therefore to a different information than the one of code \mathcal{N} .

The conclusion is that fixing to zero the difference $\mathcal{N}_{A_{\kappa}} - \mathcal{N}_{\tilde{A}_{\kappa}} = 0$, for any κ , leaves completely open the choice for the value of the code \mathcal{N} .

Thus, infinitely many memory (vacuum) states, each one of them corresponding to a different code \mathcal{N} , may exist: A huge number of sequentially recorded information data may coexist without destructive interference since infinitely many vacua $|0>_{\mathcal{N}}$, for all \mathcal{N} , are independently accessible in the sequential recording process. Recording information of code \mathcal{N}' does not necessarily produce destruction of previously printed information of code $\mathcal{N} \neq \mathcal{N}'$, contrary to the non-dissipative case. In the dissipative case the "brain (ground) state" may be represented as the collection (or the superposition) of the full set of memory states $|0>_{\mathcal{N}}$, for all \mathcal{N} . In the non-dissipative case the " \mathcal{N} -freedom" is missing and consecutive information printing produces overprinting.

The memory state is known (Vitiello, 1995) to be a two-mode coherent state (a generalized SU(1,1) coherent state) and is given, at finite volume V, by

$$|0\rangle_{\mathcal{N}} = \prod_{k} \frac{1}{\cosh \theta_k} \exp\left(-\tanh \theta_k A_k^{\dagger} \tilde{A}_k^{\dagger}\right) |0\rangle_0 ,$$
 (1)

and, for all \mathcal{N} , $\mathcal{N}\langle 0|0\rangle_{\mathcal{N}}=1$.

 $|0\rangle_{\mathcal{N}}$ is an entangled state, namely it cannot be factorized into two single–mode states. Indeed, $|0\rangle_{\mathcal{N}}$ can be written as

$$|0\rangle_{\mathcal{N}} = \left(\prod_{k} \frac{1}{\cosh \theta_{k}}\right) \left(|0\rangle_{0} \otimes |\tilde{0}\rangle_{0} - \sum_{k} \tanh \theta_{k} \left(|A_{k}\rangle \otimes |\tilde{A}_{k}\rangle\right) + \ldots\right) ,$$
 (2)

where we have explicitly expressed the tensor product between the tilde and non-tilde sector and dots stand for higher power terms. Clearly, the second factor in the right hand side of the above equation cannot be reduced to the product of two single-mode components.

We remark that the entanglement is expressed by the unitary inequivalence relation with the vacuum $|0\rangle_0 \equiv |0\rangle_0 \otimes |\tilde{0}\rangle_0$:

$$\mathcal{N}\langle 0|0\rangle_0 \underset{V\to\infty}{\longrightarrow} 0 \quad \forall \mathcal{N} \neq 0 ,$$
 (3)

which is verified only in the infinite volume limit. At finite volume, a unitary transformation could disentangle the tilde and non-tilde sectors: for a finite number of components their tensor product would be different from the entangled state. However, this is not the case in the infinite volume limit, where the summation extends to an infinite number of components. In such a limit the entanglement brain-environment is permanent. It cannot be washed out: The entanglement mathematically represents the impossibility to cut the links between the brain and the external world (a closed, i.e. fully isolated, brain is indeed a dead brain according to physiology).

Notice that memory states corresponding to different codes $\mathcal{N} \neq \mathcal{N}'$, $|0\rangle_{\mathcal{N}}$ and $|0\rangle_{\mathcal{N}'}$, are each other unitarily inequivalent in the infinite volume limit:

$$\mathcal{N}\langle 0|0\rangle_{\mathcal{N}'} \underset{V \to \infty}{\longrightarrow} 0 \quad \forall \mathcal{N} \neq \mathcal{N}'$$
 (4)

This means that there does not exist in the infinite volume limit any unitary transformation which may transform one vacuum of code \mathcal{N} into another one of code \mathcal{N}' : this fact, which is a typical feature of QFT, guarantees that the corresponding printed information data are indeed different or distinguishable ones (\mathcal{N} is a good code) and that each information printing

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is also *protected* against interference from other information printing (absence of *confusion* among information data).

The average number $\mathcal{N}_{A_{\kappa}}$ is given by

$$\mathcal{N}_{A_{\kappa}} = \mathcal{N}\langle 0|A_{\kappa}^{\dagger} A_{\kappa}|0\rangle_{\mathcal{N}} = \sinh^{2}\theta_{\kappa} , \qquad (5)$$

and relates the \mathcal{N} -set, $\mathcal{N} \equiv \{\mathcal{N}_{A_{\kappa}} = \mathcal{N}_{\tilde{A}_{\kappa}}, \forall \kappa, at \ t_0 = 0\}$ to the θ -set, $\theta \equiv \{\theta_{\kappa}, \forall \kappa, at \ t_0 = 0\}$. We also use the notation $\mathcal{N}_{A_{\kappa}}(\theta) \equiv \mathcal{N}_{A_{\kappa}}$ and $|0(\theta)\rangle \equiv |0\rangle_{\mathcal{N}}$. In general we may refer to \mathcal{N} or, alternatively and equivalently, to the corresponding θ , or viceversa.

The effect of finite (realistic) size of the system may spoil the above mentioned unitary inequivalence. In the case of open systems, in fact, transitions among (would be) unitary inequivalent vacua may occur (phase transitions) for large but finite volume, due to coupling with the external environment. The inclusion of dissipation leads thus to a picture of the system "living over many ground states" (continuously undergoing phase transitions). Note that even very weak (although above a certain threshold) perturbations may drive the system through its macroscopic configurations. In this way, occasional (random) weak perturbations are recognized to play an important rôle in the complex behavior of the brain activity.

The possibility of transitions among differently coded vacua is a feature of the model which is not completely negative: smoothing out the exact unitary inequivalence among memory states has the advantage of allowing the familiar phenomenon of the "association" of memories: once transitions among different memory states are "slightly" allowed, the possibility of associations ("following a path of memories") becomes possible. Of course, these "transitions" should only be allowed up to a certain degree in order to avoid memory "confusion" and difficulties in the process of storing "distinct" informational inputs (Vitiello, 1995; Alfinito and Vitiello, 2000). It is interesting to observe that Freeman, on the basis of experimental observations, shows that noisy fluctuations at a microscopic level may have a stabilizing effect on brain activity, noise preventing to fall into some unwanted state (attractor) and being an essential ingredient for the neural chaotic perceptual apparatus (Freeman, 1990; 1996; 2000) We will come back to consider quantum noise in Section 4.

The dwq may acquire an effective non–zero mass due to the effects of the system finite size (Vitiello, 1995; Alfinito and Vitiello, 2000). Such an effective mass will then act as a threshold for the excitation energy of dwq so that, in order to trigger the recall process, an energy supply equal or greater than such a threshold is required. When the energy supply is lower than the required threshold a "difficulty in recalling" may be experienced. At the same time, however, the threshold may positively act as a "protection" against unwanted perturbations (including thermalization) and contributes to the stability of the memory state. In the case of zero threshold any replication signal could excite the recalling and the brain would fall into a state of "continuous flow of memories" (Vitiello, 1995).

Summarizing, the brain system may be viewed as a complex system with (infinitely) many macroscopic configurations (the memory states). Dissipation, which is intrinsic to the brain dynamics, is recognized to be the root of such a complexity, namely of the huge memory capacity.

Of course, the brain has several structural and dynamical levels (the basic level of coherent condensation of dwq, the cellular cytoskeleton level, the neuronal dendritic level, and so on) which coexist, interact among themselves and influence each other's functioning. Dissipation introduces the further richness of the replicas or degenerate vacua at the basic quantum level. The crucial point is that the different levels of organization are not simply structural features of the brain, their reciprocal interaction and their evolution is intrinsically related to the basic quantum dissipative dynamics.

The brain's functional stability is ensured by the system's "coherent response" to the multiplicity of external stimuli. Thus dissipation also seems to suggest a solution to the so

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called *binding problem*, namely the understanding of the unitary response and behavior of apparently separated units and physiological structures of the brain.

When considering dwq with time–dependent frequency, modes with longer life–time are found to be the ones with higher momentum. Since the momentum is proportional to the reciprocal of the distance over which the mode can propagate, this means that modes with shorter range of propagation will survive longer. On the contrary, modes with longer range of propagation will decay sooner. The scenario becomes then particularly interesting since this mechanism may produce the formation of ordered domains of finite different sizes with different degree of stability: smaller domains would be the more stable ones. Remember now that the regions over which the dwq propagate are the domains where ordering (i.e. symmetry breakdown) is produced. Thus we arrive at the dynamic formation of a hierarchy (according to their life–time or equivalently to their sizes) of ordered domains (Alfinito and Vitiello, 2000).

4 Quantum noise

In this Section, by resorting to previous results on dissipative quantum systems (Srivastava, Vitiello and Widom, 1995; Blasone et al., 1998), we show that the doubled variables account for the quantum noise effects in the fluctuating random force in the system–environment coupling. This opens a new perspective in the quantum model of brain, and fits well with some experimental observations in the brain behavior (Freeman, 1990; 1996; 2000).

Our discussion will also show how the doubling of the degrees of freedom is related to the Wigner function and to the density matrix formalism.

Before considering dissipation, let us consider the case of zero mechanical resistance. The Hamiltonian for an isolated particle is

$$H = -\frac{\hbar^2}{2m} \left(\frac{\partial}{\partial x}\right)^2 + V(x) , \qquad (6)$$

and the expression for the Wigner function is (Feynman, 1972; Haken, 1984)

$$W(p,x,t) = \frac{1}{2\pi\hbar} \int \psi^* \left(x - \frac{1}{2}y, t \right) \psi \left(x + \frac{1}{2}y, t \right) e^{\left(-i\frac{py}{\hbar} \right)} dy . \tag{7}$$

The associated density matrix function is

$$W(x,y,t) = \langle x + \frac{1}{2}y | \rho(t) | x - \frac{1}{2}y \rangle = \psi^* \left(x - \frac{1}{2}y, t \right) \psi \left(x + \frac{1}{2}y, t \right) , \qquad (8)$$

with equation of motion given by

$$i\hbar \frac{d\rho}{dt} = [H, \rho] \ . \tag{9}$$

By introducing the notation

$$x_{\pm} = x \pm \frac{1}{2}y \;, \tag{10}$$

Eq. (9) is written in the coordinate representation as

$$i\hbar \frac{\partial}{\partial t} \langle x_{+} | \rho(t) | x_{-} \rangle = \left\{ -\frac{\hbar^{2}}{2m} \left[\left(\frac{\partial}{\partial x_{+}} \right)^{2} - \left(\frac{\partial}{\partial x_{-}} \right)^{2} \right] + \left[V(x_{+}) - V(x_{-}) \right] \right\} \langle x_{+} | \rho(t) | x_{-} \rangle, \tag{11}$$

namely, in terms of x and y, we have

$$i\hbar \frac{\partial}{\partial t} W(x, y, t) = \mathcal{H}_o W(x, y, t) ,$$
 (12)

$$\mathcal{H}_o = \frac{1}{m} p_x p_y + V\left(x + \frac{1}{2}y\right) - V\left(x - \frac{1}{2}y\right),\tag{13}$$

with $p_x=-i\hbar\frac{\partial}{\partial x},\ p_y=-i\hbar\frac{\partial}{\partial y}.$ The Hamiltonian (13) may be constructed from the Lagrangian

$$\mathcal{L}_o = m\dot{x}\dot{y} - V\left(x + \frac{1}{2}y\right) + V\left(x - \frac{1}{2}y\right). \tag{14}$$

We thus see that the density matrix and the Wigner function formalism requires the introduction of a "doubled" set of coordinates, x_{\pm} , or, alternatively, x and y.

In the case of the particle interacting with a thermal bath at temperature T, the interaction Hamiltonian between the bath and the particle is taken as

$$H_{int} = -fx, (15)$$

where f is the random force on the particle at the position x due to the bath.

In the Feynman-Vernon formalism, it can be shown (Srivastava, Vitiello and Widom, 1995) that the effective action for the particle has the form

$$\mathcal{A}[x,y] = \int_{t_i}^{t_f} dt \, \mathcal{L}_o(\dot{x}, \dot{y}, x, y) + \mathcal{I}[x,y], \tag{16}$$

where \mathcal{L}_o is defined in Eq.(14) and

$$\mathcal{I}[x,y] = \frac{1}{2} \int_{t_i}^{t_f} dt \left[x(t) F_y^{ret}(t) + y(t) F_x^{adv}(t) \right]
+ \frac{i}{2\hbar} \int_{t_i}^{t_f} \int_{t_i}^{t_f} dt ds \, N(t-s) y(t) y(s) , \qquad (17)$$

where the retarded force on y and the advanced force on x are given in terms of the retarded and advanced Greens functions and N(t-s) denotes the quantum noise in the fluctuating random force given by

$$N(t-s) = \frac{1}{2} \langle f(t)f(s) + f(s)f(t) \rangle . \tag{18}$$

The symbol $\langle ... \rangle$ denotes average with respect to thermal bath. One can then show (see (Srivastava, Vitiello and Widom, 1995)) for the details of the derivation) that the real and the imaginary part of the action are given by

$$\mathcal{R}e\mathcal{A}[x,y] = \int_{t_i}^{t_f} dt \,\mathcal{L},$$
 (19)

$$\mathcal{L} = m\dot{x}\dot{y} - \left[V(x + \frac{1}{2}y) - V(x - \frac{1}{2}y)\right] + \frac{1}{2}\left[xF_y^{ret} + yF_x^{adv}\right],\tag{20}$$

and

$$\mathcal{I}m\mathcal{A}[x,y] = \frac{1}{2\hbar} \int_{t_i}^{t_f} \int_{t_i}^{t_f} dt ds \, N(t-s)y(t)y(s). \tag{21}$$

respectively. These results, Eqs.(19), (20), and (21), are rigorously exact for linear passive damping due to the bath. They show that, in the classical limit " $\hbar \to 0$ ", nonzero y yields an "unlikely process" due to the large imaginary part of the action implicit in Eq.(21). On the contrary, at quantum level nonzero y accounts for quantum noise effects in the fluctuating

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random force in the system-environment coupling arising from the imaginary part of the action (Srivastava, Vitiello and Widom, 1995).

In the case one considers the approximation to Eq.(20) with $F_y^{ret} = \gamma \dot{y}$ and $F_x^{adv} = -\gamma \dot{x}$, and puts $V\left(x \pm \frac{1}{2}y\right) = \frac{1}{2}\kappa(x \pm \frac{1}{2}y)^2$, then the damped harmonic oscillator (dho) for the x variable and the complementary equation for the y coordinate can be derived

$$m\ddot{x} + \gamma \dot{x} + \kappa x = 0, \tag{22}$$

$$m\ddot{y} - \gamma \dot{y} + \kappa y = 0. \tag{23}$$

The y-oscillator is thus recognized to be the time-reversed image of the x-oscillator. Of course, from the manifold of solutions to Eqs. (22), (23) we could choose the ones for which the y coordinate is constrained to be zero. Then we obtain the classical damped oscillator equation from a lagrangian theory at the expense of introducing an "extra" coordinate y, later constrained to vanish.

It should be stressed, however, that the rôle of the "doubled" y coordinate is absolutely crucial in the quantum regime since there it accounts for the quantum noise in the fluctuating random force in the system-environment coupling, as shown above. Reverting, instead, from the classical level to the quantum level, the loss of information occurring at the classical level due to dissipation manifests itself in terms of "quantum" noise effects arising from the imaginary part of the action, to which the y contribution is indeed crucial.

Going back to the dissipative quantum model of brain, we note that the classical equations for the dho x and its time–reversal image y, Eqs. (22) and (23), are associated in the canonical quantization procedure to the quantum operators A and \tilde{A} (Vitiello, 1995), respectively. When we consider the quantum field theory, the A and the \tilde{A} operators get labelled by the (continuously varying) suffix κ and for each κ value we have a couple of equations of the type (22) and (23) for the field amplitudes (Celeghini, Rasetti and Vitiello, 1992).

In conclusion, we have seen that the doubling of the degrees of freedom discussed in the previous sections accounts for the quantum noise in the fluctuating random force coupling the system with the environment (the bath). On the other hand, we have also recognized the entanglement between the tilde and the non-tilde modes. Thus we conclude that brain processes are intrinsically and inextricably dependent on the quantum noise in the fluctuating random force in the brain-environment coupling. It is interesting to mention, in this respect, the rôle of noise in neurodynamics which has been noticed by Freeman in his laboratory observations (Freeman, 1990; 1996; 2000).

In the following section we discuss another feature built—in in the dissipative quantum model: the chaotic behavior of the trajectories in the space of the memory states.

5 Chaos and memory states

In this section we analyze the time evolution of the memory states.

We denote by $|0(t)\rangle_{\mathcal{N}}$ the memory state at time t and refer to the space of the memory states, for all \mathcal{N} and at any time t, as to the "memory space". In this space the memory state $|0(t)\rangle_{\mathcal{N}}$ may be thought as a "point" labelled by a given \mathcal{N} -set (or θ -set) and by a given value of t and points corresponding to different \mathcal{N} (or θ) sets and different t's are distinct points (do not overlap, cf. Eq. (4) and Eq. (27) below). As mentioned, the memory states can be understood as the vacuum states of corresponding Hilbert spaces. In QFT these Hilbert spaces are denoted as the representations of the canonical commutation relations for the operators A and \tilde{A} , which in the infinite volume limit are unitarily inequivalent. So, the memory space may also be denoted as the "space of the (unitarily inequivalent) representations", each representation being represented by a "point" in such a larger memory space.

We show that trajectories over the memory space (the representation space) may be chaotic trajectories.

The requirements for chaotic behavior in non–linear dynamics can be formulated as follows (Hilborn, 1994):

- i) the trajectories are bounded and each trajectory does not intersect itself (trajectories are not periodic).
 - ii) there are no intersections between trajectories specified by different initial conditions.
 - iii) trajectories of different initial conditions are diverging trajectories.

At finite volume V, the memory state $|0(t)\rangle_{\mathcal{N}}$, to which the memory state, say at $t_0 = 0$, $|0\rangle_{\mathcal{N}}$ evolves, is given (Vitiello, 1995) by

$$|0(t)\rangle_{\mathcal{N}} = \prod_{\kappa} \frac{1}{\cosh\left(\Gamma_{\kappa}t - \theta_{\kappa}\right)} \exp\left(\tanh\left(\Gamma_{\kappa}t - \theta_{\kappa}\right)A_{\kappa}^{\dagger}\tilde{A}_{\kappa}^{\dagger}\right)|0\rangle_{0} , \qquad (24)$$

which also is an entangled, SU(1,1) generalized coherent state. Γ_{κ} is the damping constant implied by the dissipation (Vitiello, 1995). Note that for any t

$$\mathcal{N}\langle 0(t)|0(t)\rangle_{\mathcal{N}} = 1. \tag{25}$$

In the infinite volume limit we have (for $\int d^3 \kappa \Gamma_{\kappa}$ finite and positive)

$$\mathcal{N}\langle 0(t)|0\rangle_{\mathcal{N}} \underset{V\to\infty}{\longrightarrow} 0 \quad \forall t \quad ,$$
 (26)

$$\mathcal{N}\langle 0(t)|0(t')\rangle_{\mathcal{N}}\underset{V\to\infty}{\longrightarrow} 0 \quad \forall t,t' \quad , \quad t\neq t' \quad .$$
 (27)

States $|0(t)\rangle_{\mathcal{N}}$ (and the associated Hilbert spaces $\{|0(t)\rangle_{\mathcal{N}}\}$) at different time values $t \neq t'$ are thus unitarily inequivalent in the infinite volume limit.

Time evolution of the memory state $|0\rangle_{\mathcal{N}}$ is thus represented as the (continuous) transition through the representations $\{|0(t)\rangle_{\mathcal{N}}\}$ at different t's (same \mathcal{N}), namely by the "trajectory" through the "points" $\{|0(t)\rangle_{\mathcal{N}}\}$ in the space of the representations. The trajectory "initial condition" at $t_0=0$ is specified by the \mathcal{N} -set.

It is known (Manka, Kuczynski and Vitiello, 1986; Del Giudice et al., 1988) (see also (Vitiello, 2003) for a recent discussion) that trajectories of this kind are classical trajectories: transition from one representation to another inequivalent one would be strictly forbidden in a quantum dynamics.

We now observe that the trajectories are bounded in the sense of Eq. (25), which shows that the "length" of the "position vectors" (the state vectors at time t) in the representation space is finite (and equal to one) for each t (by resorting to the properties of the SU(1,1) group, one can show that the set of points representing the coherent states $|0(t)\rangle_{\mathcal{N}}$ for any t is isomorphic to the union of unit circles of radius $r_{\kappa}^2 = \tanh^2(\Gamma_{\kappa}t - \theta_{\kappa})$ for any κ (Perelomov 1986; Pessa and Vitiello, 2003).

We also note that Eqs. (26) and (27) express the fact that the trajectory does not crosses itself as time evolves (it is not a periodic trajectory): the "points" $|0(t)\rangle_{\mathcal{N}}$ and $|0(t')\rangle_{\mathcal{N}}$ through which the trajectory goes, for any t and t', with $t \neq t'$, after the initial time $t_0 = 0$, never coincide. The requirement i) is thus satisfied.

In the infinite volume limit, Eqs. (26) and (27) also hold for $\mathcal{N} \neq \mathcal{N}'$, i.e. we also have

$$\mathcal{N}\langle 0(t)|0\rangle_{\mathcal{N}'} \underset{V\to\infty}{\longrightarrow} 0 \quad \forall t \quad , \quad \forall \mathcal{N} \neq \mathcal{N}'$$
 (28)

$$\mathcal{N}\langle 0(t)|0(t')\rangle_{\mathcal{N}'} \underset{V\to\infty}{\longrightarrow} 0 \quad \forall t, t' \quad , \quad \forall \mathcal{N} \neq \mathcal{N}' .$$
 (29)

The derivation of Eqs. (28) and (29) rests on the fact that in the continuum limit, for given t and t' and for $\mathcal{N} \neq \mathcal{N}'$, $\cosh(\Gamma_{\kappa}t - \theta_{\kappa} + \theta'_{\kappa})$ and $\cosh(\Gamma_{\kappa}(t - t') - \theta_{\kappa} + \theta'_{\kappa})$, respectively, are never identically equal to one for all κ .

Notice that Eq. (29) is true also for t = t' for any $\mathcal{N} \neq \mathcal{N}'$. Eqs. (28) and (29) thus tell us that trajectories specified by different initial conditions ($\mathcal{N} \neq \mathcal{N}'$) never cross each other. Thus, also requirement ii) is satisfied.

We remark that, in the infinite volume limit, due to the property ii) no *confusion* (interference) arises among different memories, even as time evolves. In realistic situations of finite size coherent domains, some *association* of memories may become possible. In such a case, this means the at a "crossing" point between two, or more than two, trajectories, from one of these trajectories one can switch to another one which there crosses. This may be felt indeed as association of memories or as "switching" from one information to another one.

The average number of modes of type A_{κ} is given, at each instant t, by

$$\mathcal{N}_{A_{\kappa}}(\theta, t) \equiv \mathcal{N}\langle 0(t)|A_{\kappa}^{\dagger} A_{\kappa}|0(t)\rangle_{\mathcal{N}} = \sinh^{2}(\Gamma_{\kappa} t - \theta_{\kappa}) , \qquad (30)$$

and similarly for the modes of type \tilde{A}_{κ} . This number can been shown to satisfy the Bose distribution and thus it is actually a statistical average (Vitiello, 1995). From Eq. (30) we see that at a time $t = \tau$, with τ the largest of the values $t_{\kappa} \equiv \frac{\theta_{\kappa}}{\Gamma_{\kappa}}$, the memory state $|0\rangle_{\mathcal{N}}$ is reduced (decayed) to the "empty" vacuum $|0\rangle_0$: the information has been forgotten, the \mathcal{N} code is decayed. The time $t = \tau$ can be taken to be the life–time of the memory of code \mathcal{N} (for details on this point we refer to (Alfinito and Vitiello, 2000), where a detailed analysis of the life–time of the κ –modes has been made). We conclude that the time evolution of the memory state leads to the "empty" vacuum $|0\rangle_0$ which acts as a sort of attractor state. However, as time goes on, i.e. as t gets larger than τ , we have

$$\lim_{t \to \infty} \mathcal{N}\langle 0(t)|0\rangle_0 \propto \lim_{t \to \infty} \exp\left(-t\sum_{\kappa} \Gamma_{\kappa}\right) = 0 , \qquad (31)$$

which tells us that the state $|0(t)\rangle_{\mathcal{N}}$ "diverges" away from the attractor state $|0\rangle_0$ with exponential law (we are always assuming $\sum_{\kappa} \Gamma_{\kappa} > 0$).

It is interesting to observe that in order to avoid to fall into such an attractor, i.e. in order to not forget certain information, one needs to "restore" the \mathcal{N} code by "refreshing" the memory by brushing up the subject (external stimuli maintained memory). This means that one has to recover the whole \mathcal{N} -set (if the whole code is "corrupted"), or "pieces" of the memory associated to those \mathcal{N}_{κ} , for certain κ 's, which have been lost at $t_{\kappa} = \frac{\theta_{\kappa}}{\Gamma_{\kappa}}$. The operation of restoring the code appears to be a sort of "updating the register" of the memories since it amounts to reset the memory code (and clock) to the (updated) initial time t_0 . We also observe that even after the time τ is passed by, the code \mathcal{N} may be recovered provided t is not much larger than τ (namely, as far as the approximation of $\cosh(\Gamma_{\kappa}t - \theta_{\kappa}) \approx \exp(-t\sum_{\kappa}\Gamma_{\kappa})$ does not hold, cf. Eq. (31)).

We now consider the variation in time of the "distance" between trajectories in the memory space, i.e. the variation in time of the difference between two different codes, $\mathcal{N} \neq \mathcal{N}'$ $(\theta \neq \theta')$, corresponding to different initial conditions for two trajectories. At time t, each component $\mathcal{N}_{A_{\kappa}}(t)$ of the code $\mathcal{N} \equiv \{\mathcal{N}_{A_{\kappa}} = \mathcal{N}_{\tilde{A}_{\kappa}}, \forall \kappa, at \ t_0 = 0\}$ is given by the expectation value in the memory state of number operator $A_{\kappa}^{\dagger}A_{\kappa}$. The difference is then (cf. Eq. (30):

$$\Delta \mathcal{N}_{A_{\kappa}}(t) \equiv \mathcal{N}'_{A_{\kappa}}(\theta', t) - \mathcal{N}_{A_{\kappa}}(\theta, t) =$$

$$= \sinh^{2}(\Gamma_{\kappa}t - \theta_{\kappa} + \delta\theta_{\kappa}) - \sinh^{2}(\Gamma_{\kappa}t - \theta_{\kappa}) \approx \sinh(2(\Gamma_{\kappa}t - \theta_{\kappa}))\delta\theta_{\kappa} , \qquad (32)$$

where $\delta\theta_{\kappa} \equiv \theta_{\kappa} - \theta'_{\kappa}$ (which without loss of generality may be assumed to be greater than zero), and the last equality holds for small $\delta\theta_{\kappa}$ (i.e. for a very small difference in the initial conditions of the two memory states). The time–derivative then gives

$$\frac{\partial}{\partial t} \Delta \mathcal{N}_{A_{\kappa}}(t) = 2\Gamma_{\kappa} \cosh(2(\Gamma_{\kappa} t - \theta_{\kappa})) \delta \theta_{\kappa} , \qquad (33)$$

which shows that the difference between $\mathcal{N}_{A_{\kappa}}$'s, originally even slightly different, is a growing function of time. For enough large t the modulus of the difference $\Delta \mathcal{N}_{A_{\kappa}}(t)$ and its variation in time diverge as $\exp(2\Gamma_{\kappa}t)$, for all κ 's. For each κ , $2\Gamma_{\kappa}$ play thus the rôle similar to the one of the Lyapunov exponent.

Thus we conclude that trajectories in the memory space differing by a small variation $\delta\theta$ in the initial conditions are diverging trajectories as time evolves.

It should be remarked that, as shown by Eq. (32), the difference between specific κ -components of the codes \mathcal{N} and \mathcal{N}' may become zero at a given time $t_{\kappa} = \frac{\theta_{\kappa}}{\Gamma_{\kappa}}$. However, this does not mean that the difference between the codes \mathcal{N} and \mathcal{N}' becomes zero. They are made, indeed, by a large number (infinite number, in the continuum limit) of components and they are still different codes even if a finite number of their components are equal. On the contrary, always for very small $\delta\theta_{\kappa} \equiv \theta_{\kappa} - \theta'_{\kappa}$, suppose that the time interval $\Delta t = \tau_{max} - \tau_{min}$, with τ_{min} and τ_{max} the smallest and the largest, respectively, of the $t_{\kappa} = \frac{\theta_{\kappa}}{\Gamma_{\kappa}}$, for all κ 's, is "very small", then in such a Δt the codes are "recognized" to be "almost" equal. In such a case, Eq. (32) expresses the "recognition" (or recall) process and we see how it is possible that "slightly different" $\mathcal{N}_{A_{\kappa}}$ -patterns (or codes) are "identified" (recognized to be the "same code" even if corresponding to slightly different inputs). Roughly, Δt may be taken as a measure of the "recognition time".

We finally recall that $\sum_{\kappa} E_{\kappa} \dot{\mathcal{N}}_{A_{\kappa}} dt = \frac{1}{\beta} dS_A$ (see (Vitiello, 1995)), where E_{κ} is the energy of the mode A_{κ} , $\beta = \frac{1}{k_B T}$, k_B the Boltzmann constant, dS_A is the entropy variation associated to the modes A and $\dot{\mathcal{N}}_{A_{\kappa}}$ denotes the time derivative of $\mathcal{N}_{A_{\kappa}}$. Eq. (33) then leads to the relation between the differences in the variations of the entropy and the divergence of trajectories of different initial conditions:

$$\Delta \sum_{\kappa} E_{\kappa} \dot{\mathcal{N}}_{A_{\kappa}}(t) dt = \sum_{\kappa} 2E_{\kappa} \Gamma_{\kappa}^{2} \cosh(2(\Gamma_{\kappa} t - \theta_{\kappa})) \delta \theta_{\kappa} dt = \frac{1}{\beta} (dS_{A}' - dS_{A}) . \tag{34}$$

In conclusion, also the requirement iii) is satisfied.

We thus conclude that the trajectories in the memory space may exhibit chaotic behavior. This is a feature which fits experimental observations by Freeman (Freeman, 1990; 1996; 2000) who indeed finds characteristic chaotic behavior in neural aggregates of the olfactory system of laboratory pets.

6 Concluding remarks

We have presented a short review of the dissipative quantum model of brain and we have shown that the doubling of the system degrees of freedom account for the quantum noise in the fluctuating random force coupling the system with the environment. Due to the permanent entanglement brain—environment, quantum noisy effects are intrinsically present in the brain dynamics. Moreover, we have seen that time evolution in the memory space may present chaotic behavior. It is interesting to remark that laboratory observations show an important rôle in brain dynamics of noise and of chaos (Freeman, 1990; 1996; 2000). In the dissipative model noise and chaos turn out to be natural ingredients of the model. In particular the chaotic behavior of the trajectories in memory space may account of the high perceptive resolution in the recognition of the inputs. Indeed, small differences in the codes associated to external inputs may lead to diverging differences in the corresponding memory paths. On the other side, codes "almost" equal in all of their components may easily be recognized as being the "same" code (code identification, or "code—pattern" recognition).

Further work on these subjects is in progress (Pessa and Vitiello, 2003).

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